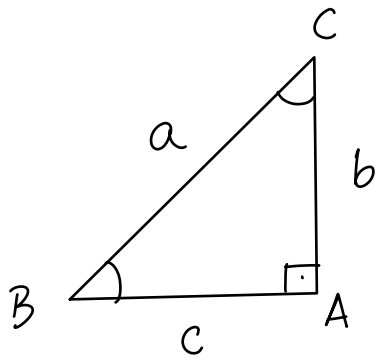
Funções trigonométricas

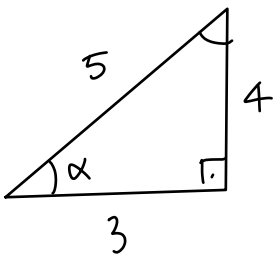


$$\cos \hat{B} = \frac{c}{a} \qquad \cos \hat{C} = \frac{b}{a}$$

$$\operatorname{sen} \hat{B} = \frac{b}{a} \qquad \operatorname{sen} \hat{C} = \frac{c}{a}$$

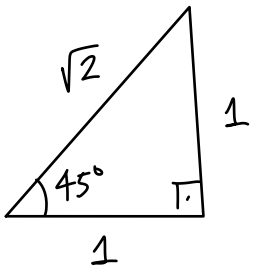
Pitágoras: $a^2 = b^2 + c^2$

Exemplo:



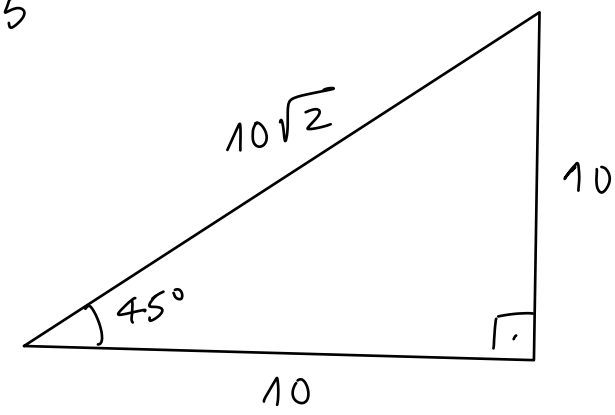
$$\cos \alpha = \frac{3}{5}$$

$$\operatorname{sen} \alpha = \frac{4}{5}$$



$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\operatorname{sen} 45° = \frac{1}{\sqrt{2}}$$



Não importa o tamanho do triângulo!
Depende apenas do ângulo.

Observe que:

$$(\operatorname{sen} \hat{B})^2 + (\cos \hat{B})^2 = \left(\frac{b}{a}\right)^2 + \left(\frac{c}{a}\right)^2 = \frac{b^2}{a^2} + \frac{c^2}{a^2}$$

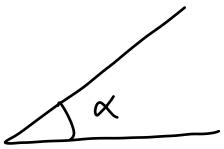
$$= \frac{b^2 + c^2}{a^2} = \frac{a^2}{a^2} = 1$$

$$(\operatorname{sen} \alpha)^2 + (\cos \alpha)^2 = 1$$

$$\boxed{\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1}$$

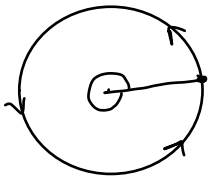
$$\cos^2 \alpha = 1 - \operatorname{sen}^2 \alpha$$

$$\operatorname{sen}^2 \alpha = 1 - \cos^2 \alpha$$



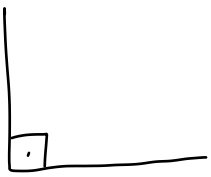
graus

radianos



360°

2π rad



90°

$\frac{\pi}{2}$ rad

180°

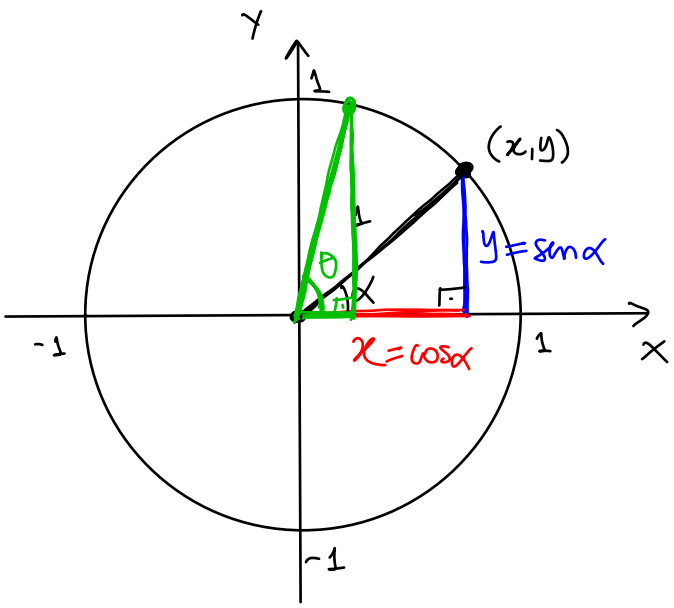
π rad

Exemplos: 30°: $\frac{30^\circ}{x} = \frac{360^\circ}{2\pi} \Rightarrow 360x = 60\pi \Rightarrow x = \frac{60\pi}{360} = \frac{\pi}{6}$.

$\frac{\pi}{4}$ rad: $\frac{x}{\pi/4} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi} = \frac{90^\circ}{\pi/2} \Rightarrow \pi x = \frac{180\pi}{4} \Rightarrow x = 45^\circ$

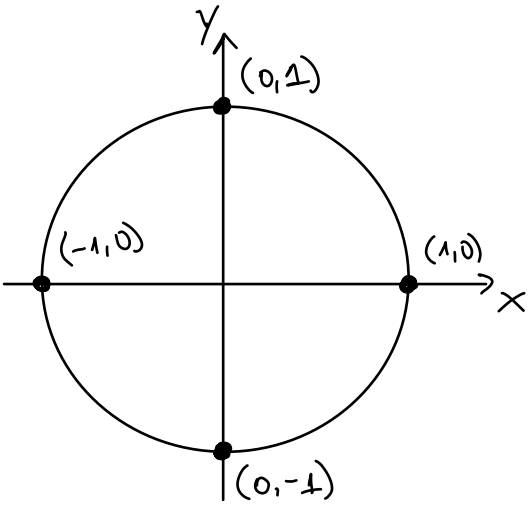
$\frac{\pi}{4}$ "=" $\frac{180}{4} = 45$, $30 = \frac{180}{6}$ "=" $\frac{\pi}{6}$

Funções: $\cos: \mathbb{R} \to \mathbb{R}$, $\operatorname{sen}: \mathbb{R} \to \mathbb{R}$



$\cos\alpha = \frac{x}{1} = x$

$\operatorname{sen}\alpha : \frac{y}{1} = y$



α	cos	sen
0	1	0
90°, $\pi/2$	0	1
180°, π	-1	0
270°, $\frac{3\pi}{2}$	0	-1
360°, 2π	1	0

α	cos	sen
30°, $\pi/6$	$\sqrt{3}/2$	$1/2$
45°, $\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
60°, $\pi/3$	$1/2$	$\sqrt{3}/2$

Outras funções:

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}, \quad \sec\alpha = \frac{1}{\cos\alpha}, \quad \operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha}, \quad \operatorname{cotg}\alpha = \frac{\cos\alpha}{\operatorname{sen}\alpha}$$

$$\cos\alpha = 0 \Rightarrow \alpha \in \left\{\ldots, -\tfrac{5\pi}{2}, -\tfrac{3\pi}{2}, -\tfrac{\pi}{2}, \tfrac{\pi}{2}, \tfrac{3\pi}{2}, \tfrac{5\pi}{2}, \tfrac{7\pi}{2}, \ldots\right\}$$

$\therefore \alpha = (2k+1)\frac{\pi}{2}, k \in \mathbb{Z}$ (múltiplos ímpares de $\frac{\pi}{2}$)

$$\operatorname{sen}\alpha = 0 \Rightarrow \alpha \in \{\ldots, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi, \ldots\}$$

$\therefore \alpha = k\pi, k \in \mathbb{Z}$ (múltiplos de π)